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Desensitizing Structural-Control Design

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A method for targeted robustness optimization is presented. This method allows the analyst to target the most critical performance parameter for enhanced protection from uncertainties in the model. To create this protection, this method uses statistical data about the uncertainties in the parameters of the model. The power of the targeted robustness method is demonstrated numerically for the active damping of two truss examples. The results show that the sensitivity of the most critical damping ratio to uncertainties is far below those of noncritical damping ratios.

Introduction

LARGE civil and space structures planned for the next century are expected to depend on active control systems to achieve a variety of structural response objectives. Examples include tall buildings whose response to wind loads and earthquakes is actively controlled and large space antennas designed to achieve precise pointing using active shape and vibration control.

Control systems are designed based on models of both the structure to be controlled and the control hardware, which includes sensors, actuators, and analog or digital computers. There may be significant differences between the models and the real systems. One reason is modeling deficiencies such as neglect of nonlinearities or truncation of the model. Another reason is variability in material properties, geometrical dimensions, and the construction and assembly processes.

Accuracy of structural models is often improved in an iterative process that involves testing the structure and comparing the response to a numerical simulation based on the model. There are three problems associated with this approach. First, it is difficult to test large civil and space structures due to their size. Second, it is difficult to predict actual in-orbit performance of large space structures based on ground tests. Finally, testing the structure to improve the model is costly due to the actual expense of the tests as well as the cost associated with delays in the design process while the structure is being tested.

There are several approaches to developing active control systems based on inaccurate models. One solution is to use adaptive control techniques.[1] With this approach the model is being identified while the structure is being controlled, and the control gains are adjusted according to this identified model. Another solution is to design robust control systems that do not depend on accurate structural models for adequate performance. Examples of these techniques include H_∞ designs,[2,3] restoring robustness to the well-known linear-quadratic Gaussian (LQG) control (e.g., using loop-transfer recovery[4]), optimizing various measures of system robustness while satisfying limits on the eigenvalues or assigning them to desired locations,[5–7] or using very simple and inherently robust control designs such as collocated direct-rate feedback control.[8]

The previously mentioned techniques provide blanket robustness, or robustness to unstructured uncertainties. That is, there is no attempt to capitalize on knowledge of the nature of the uncertainties for each parameter, such as statistical properties, or on knowledge of the particular vulnerabilities of the system to be designed (such as knowledge that a certain mode of vibration is likely to be the most difficult to control). In the past few years there has been growing interest in robustness techniques that capitalize on such knowledge—techniques that are called here targeted robustness methods. Robustness can be targeted by tailoring the design to known statistical properties of the uncertainties in the system or by focusing on shielding selected (target) response quantities from the effect of these uncertainties. This targeted robustness approach can benefit from recent interest in statistically based structural dynamics models.[9]

Targeted robustness methods allow the choice of structural parameters as well as control system gains to enhance robustness. Also, targeted robustness approaches can make use of knowledge of uncertainties in the structural model. Grandhi et al.[10] considered optimizing structural and control parameters to improve a robustness measure due to Juang et al.[11] One attractive feature of that robustness measure is that it allows the structural analyst to assign different levels of uncertainty to the different elements of the structural matrices used in the control design, reflecting the differing levels of uncertainties in different physical parameters. Junkins and Rew[12] and Lim and Junkins[13] designed structural and control parameters to minimize the sensitivity of the eigenvalues of the closed-loop system to critical parameters. This approach allows the targeting of the most critical eigenvalue for protection from uncertainties. Thomas and Schmit[14] optimized structural parameters to shield the most critical constraints from the worst combination of uncertainties in the values of the design variables of the structure and control system. The objective of the present work is to develop a method of targeted robustness that shields the most critical response quantities from the cumulative effect of uncertainties in some parameters of the system.

In this paper, the targeted robustness method will be derived. We will first show how the robustness can be targeted on the most critical response quantity, and we will define the Kresselmeier-Steinhauser (KS) function used to express it mathematically. We will then focus on the special case of the control of truss structures using active members. A linear integer programming formulation for solving the actuator location problem will be presented. The targeted robustness design procedure will then be applied to the special case of protecting the most critical damping ratio of a controlled truss. Results from two numerical examples are then presented which demonstrate the power of this method.

Targeted Robustness Design Formulation

Minimization of the Most Critical Standard Deviation

The first premise of targeted robustness formulations is that the analyst can obtain some statistics of the uncertainties

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associated with various parameters of the structure and control system, such as geometrical dimensions, stiffness values, mass properties, and control system gains. The present state of the art is such that detailed statistical properties of these parameters are usually not available, but it is expected that both the nominal values of these parameters and some rough estimates of their standard deviations should be available to the designer. Although the technique described here is more general in nature, the following derivations assume that the statistical behavior of the parameters can be reasonably approximated by the normal distribution.

Consider a response quantity $r(p)$, a function of vector p of problem parameters, which have a nominal value of p_0 and some known statistical distribution, with $r_0 = r(p_0)$. We can find the statistical properties of r by Monte Carlo simulation. However, some statistical measures of r can be easily estimated directly. To do this, approximate r by a first-order Taylor series as

$$r(p) = r_0 + \nabla r^T \Delta p \tag{1}$$

where ∇r denotes the gradient of r and

$$\Delta p = p - p_0 \tag{2}$$

An approximation $V(r)$ to the variance of r (square of the standard deviation) is

$$V(r) = E[(r - r_0)^2] = \nabla r^T E(\Delta p \Delta p^T) \nabla r \tag{3}$$

where E denotes the expected value operator, and $E(\Delta p \Delta p^T)$ is the covariance matrix of the uncertainties. The diagonal terms of this covariance matrix are the squares of the standard deviations of the components of p, which we assume can be estimated. The off-diagonal entries in the covariance matrix measure the strength of the correlation between the uncertainties in the components.

The targeted robustness procedure designs the structure and control system to satisfy performance requirements while minimizing the variation due to parameter uncertainty of the most critical performance requirement. In the design procedure we have the freedom to change some of the system parameters, while others stay fixed. Denoting p_d as the subset of p, which consists of design variables, the design problem can be formulated as

$$\min_{p_d} \; V(c_m) \tag{4a}$$

such that

$$c_j(p) \leq 0, \qquad j = 1, \ldots, n_c \tag{4b}$$

where c_j represent n_c performance constraints such as limits on damping ratios, or design limits such as upper bounds on gains, and c_m is the most critical (most positive) constraint. There are some computational problems associated with the solution of such an optimization problem because the identity of the most critical constraint changes as we search the design space. Also, if there are several constraints that are all as critical or nearly as critical, then we would want to minimize the variation V for all of them. These problems are solved by the use of an envelope function, discussed in the next section, that replaces the individual constraints.

The design obtained by the solution of Eq. (4) satisfies both aspects of targeted robustness. First, it takes advantage of any knowledge of the statistical nature of the uncertainties available to the designer. Such knowledge need not be complete. As long as there is some knowledge of the statistics of important sources of uncertainties, the solution of Eq. (4) will endow the design with robustness with respect to these uncertainties. The solution will not affect robustness with respect to other uncertainties, and for this reason it is desirable to use design techniques that help with unstructured uncertainties. Also, protection against the effect of inaccuracies associated with model reduction or nonlinearities is not addressed by the targeted robustness approach. Thus, targeted robustness is not intended to be a replacement for techniques such as H_∞ design or spillover alleviation methods,[15] but as a supplemental procedure.

The design obtained by the solution of Eq. (4) also satisfies the second aspect of targeted robustness—the most critical response quantities are targeted for extra protection.

Kresselmeier-Steinhauser Envelope Function

The Kresselmeier-Steinhauser (KS) function[16,17] was developed to collapse a number of constraints into a single (envelope) constraint. Consider a vector of design parameters (such as structural dimensions and control gains) x, and a set of n_c constraints written as

$$c_j(x) \leq 0, \qquad j = 1, \ldots, n_c \tag{5}$$

In active structural control problems, a typical constraint is

$$c_j = \zeta_j^R - \zeta_j \leq 0 \tag{6}$$

where ζ_j is the damping ratio associated with the jth mode, and ζ_j^R is the required damping ratio. The KS function replaces these constraints with a single constraint

$$\mathrm{KS}(c_j) = (1/\rho)\, ln \left[\sum_{j=1}^{nc} e^{\rho c_j} \right] \leq 0 \tag{7}$$

Because of the exponential sum in the KS function, for large values of ρ the function $\mathrm{KS}(c_j)$ is virtually equal to the most critical (most positive) constraint. Since the identity of the most positive constraint changes with changes in the design x, the most critical constraint is not a smooth function of the design. The KS function is a smooth envelope of c_j, but for large values of ρ it can have very large curvatures. For this reason, we usually work with moderate values of ρ, for which the envelope function is slightly more stringent than the constraints. We can use the standard deviation of the KS function as an approximation to the standard deviation of the most critical constraint. When that constraint is a limit on the damping ratio, such as Eq. (6), this is equal to the standard deviation of the most critical damping ratio. Thus, by minimizing the standard deviation of the KS function we can minimize the standard deviation of the most critical damping ratio without knowing a priori the identity of this damping ratio.

In many cases we would be interested in minimizing not only the standard deviation of the most critical damping ratio, but also the standard deviations of all near-critical damping ratios. This can also be accommodated by using a moderate value of ρ. As ρ is reduced the distinction between critical and near-critical constraints becomes more blurred, and the standard deviation of the KS function reflects contributions from all constraints that are within a certain band of the most critical one.

We have just presented the basis of the targeted robustness optimization procedure. We will now show how this method is implemented in the case of truss structures controlled with active members. The objective in this class of problems will be to satisfy minimum requirements on the damping ratios of selected modes and to minimize the sensitivity of the most critical of these requirements by adjusting a selection of structural and control parameters.

Application: Control of a Truss Structure with Active Members

In the present work we consider the targeted robustness design of a truss with a number of active members, using decentralized control. Since we are dealing with trusses controlled with active members, it is clear that the locations of these active members influence both the performance and the robustness of the design. Ideally, they would be included as design variables in the overall optimization. However, actuator locations in a truss being discrete variables in nature, they are not handled easily with conventional gradient-based optimization algorithms. For that reason, and because our purpose is merely to establish the feasibility of the targeted robustness approach, we separate the problem into two successive optimization phases: first, we optimize actuator locations for maximum performance using fixed values for the controller gains, then we tune the structure and the control gains with fixed actuator locations to improve robustness.

In the next sections, we introduce the active member and its control system and develop the mathematical model of the controlled structure. We then show that the first phase of the optimization—finding optimal locations for the active members—can be solved using a linear programming formulation. We then describe the formulation of the second phase of the optimization—the minimization of the standard deviation of the most critical damping ratio.

Active Member

Each active member consists of a piezoelectric stack to provide actuation, and a force transducer to provide sensing. The actuator responds to an applied voltage V by a change δ in its natural length

$$\delta = g_a V \tag{8}$$

where g_a is an actuator gain. The force sensor produces a voltage signal v given as

$$v = g_s p_e = g_s k_a(\Delta - \delta) \tag{9}$$

where g_s is a sensor gain, p_e is the force in the member, k_a is the element spring constant, Δ is the total elongation of the element, and δ is the elongation due to any applied voltage. The control law used here (after Refs. 18 and 19) relates the applied voltage to the sensor voltage as

$$V = \int g_c v(t)\, \mathrm{d}t \tag{10}$$

where g_c is an adjustable control gain.

The closed-loop description of the active member (see Fig. 1) is therefore

$$\delta = \int g k_a(\Delta - \delta)\, \mathrm{d}t, \qquad g = g_s g_a g_c \tag{11}$$

Equations of Motion and Analysis Technique

For a truss with ℓ degrees of freedom and m active members, a mass matrix $M(\ell \times \ell)$ and a stiffness matrix $K(\ell \times \ell)$, the equations of motion (neglecting inherent damping) are written as

$$M\ddot{u} + Ku = f + Bp = f + BK_a\delta \tag{12}$$

where a dot denotes differentiation with respect to time, $u(\ell \times 1)$ is the nodal displacement vector, $f(\ell \times 1)$ is an applied force vector, $B(\ell \times m)$ is an influence matrix containing directional cosines of the active members, $p(m \times 1)$ is a vector of

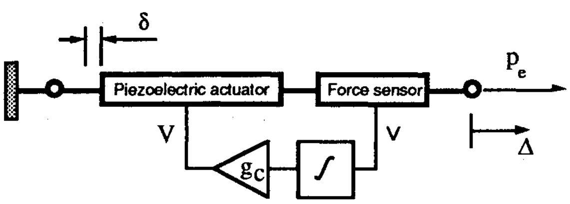

Fig. 1 Active member with integral force feedback.

active member forces, $K_a(m \times m)$ is a diagonal matrix containing the spring constants k_a of the active members, and $\delta(m \times 1)$ is a vector of applied elongations δ in the active members. Differentiating Eq. (11) with respect to time and writing it for all active members we obtain

$$\dot{\delta} = GK_a(B^T u - \delta) \tag{13}$$

where $G(m \times m)$ is a diagonal matrix containing the gains g of the m active members. We now assume that there are no applied forces, and write Eqs. (12) and (13) in state-space form as

$$\dot{x} = Ax \tag{14}$$

where

$$x = \begin{Bmatrix} u \\ \dot{u} \\ \delta \end{Bmatrix}, \qquad A = \begin{bmatrix} 0 & I & 0 \\ -M^{-1}K & 0 & M^{-1}BK_a \\ GK_aB^T & 0 & -GK_a \end{bmatrix} \tag{15}$$

In the optimization, the computation of the constraints and objective function requires the repetitive solution of the eigenvalue problem for A. For structures with a large number of degrees of freedom, this can entail a massive computational effort, and therefore we employ an approximation for estimating the eigenvalues of the matrix A. We found that the usual modal reduction technique used in structural analysis can be very inaccurate in predicting the damping ratios of structures with active members. The explanation lies in the local character of the actuation. In Ref. 20 we show that with the use of Ritz vectors for the static response to actuation forces in addition to mode shapes in the reduced basis, the accuracy of the approximation is improved by up to three orders of magnitude (also see Ref. 21).

For the calculation of the standard deviations of the damping ratios, we need the derivatives of the eigenvalues with respect to uncertainty parameters. Furthermore, for the optimization of the most critical standard deviation we need derivatives of the standard deviation, which amounts to a need for second derivatives of the eigenvalues. This creates the need for a very efficient procedure for evaluating these derivatives. Reference 20 shows how an efficient analytical expression can be derived for the sensitivity derivatives by taking advantage of symmetries in the eigensystem. The Ritz model mentioned above is used to evaluate approximate eigenvectors to be used in that expression.

Optimum Actuator Locations

The first step in the optimization is the selection of sites for active members. For this purpose, we use a first order approximation of the damping ratios of a structure controlled with active members.[18,19] Using a truncated Taylor series for the kth eigenvalue λ_k of the controlled structure, we have

$$\lambda_k(G) \approx \lambda_k(0) + \sum_{i=1}^{m} g_i \left. \frac{\partial \lambda_k}{\partial g_i} \right|_{G=0} \tag{16}$$

where $\lambda_k(G)$ is the approximate eigenvalue for some nonzero values of the active member gains and $\lambda_k(0)$ is the corresponding eigenvalue of the uncontrolled structure (zero gains, $G = 0$). The sum in the right-hand side extends to all m active members in the structure. Notice that the derivatives are taken in the uncontrolled state (zero gains). These derivatives can be evaluated using the left and right eigenvectors of the uncontrolled structure which can in turn be expressed in terms of the mode shapes. The damping ratios can then be extracted from the eigenvalues as[20]

$$\zeta_k \approx \frac{1}{2\omega_k} \sum_{i=1}^{m} g_i \nu_{ik} k_{a_i} \tag{17}$$

where ω_k and ζ_k are the natural frequency and damping ratio of mode number k, g_i is the gain of active member number i, and ν_{ik} is the fraction of the elastic energy of mode number k in the active member number i. These can be evaluated from the mode shapes of the uncontrolled structure. Equation (17) is linear in the gains of the active members and gives a very inexpensive approximation of the damping ratios for any location of the active members.

The gains of the active members are set to a nominal value. Each truss member is assigned a zero-one design variable e_i, $i = 1, \ldots, n$ where n is the number of bars in the structure. Then $e_i = 1$ if member i is an active member, and $e_i = 0$ if the member is not active. These variables are then optimized to produce the biggest margin β over the required damping ratio ζ_k^R by solving the following optimization problem:

Find e_i, $i = 1, \ldots, n$ and β

to maximize β

$$\text{such that } \zeta_k = \frac{1}{2\omega_k}\sum_{i=1}^{n} g_i e_i \nu_{ik} k_{a_i} \geq \zeta_k^R + \beta$$

$$\text{and } \sum_{i=1}^{n} e_i = m \tag{18}$$

where the gains g_i are fixed to nominal values. The last constraint in Eq. (18) requires that there are exactly m active members. Problem (18) is a linear integer programming problem, and it is solved by the branch and bound technique using the LINDO program.[22]

Optimization for Robustness of Critical Damping Ratio

Next the standard deviation of the most critical constraint is optimized by varying control gains g and structural parameters s. The problem can be restated, rewriting Eq. (4) with the KS function, Eq. (7), and the parameters relevant to a truss,

$$\min_{g,s} V[\mathrm{KS}(c_k)]$$

such that

$$c_k(g,s,\epsilon) = \zeta_k^R - \zeta_k \leq 0, \qquad k = 1, \ldots, n_{\text{modes}} \tag{19}$$

where g, s, and ϵ stand for the gains of the active members, the structural design variables, and the uncertainties in the model, respectively, n_{modes} is the number of target modes and $V(\mathrm{KS})$ is the variance of the KS function. As explained before, the KS function is a close approximation to the most critical constraint.

The optimization was performed with the NEWSUMT-A program,[23] which is based on an extended interior penalty function with approximate second derivatives and Newton's method for unconstrained minimization.

To demonstrate the targeted robustness approach, the standard deviation of the most critical damping ratio was minimized for two truss examples. Minimum values for the damping ratios of the first few modes were selected, and constraints in the form of Eq. (6) were imposed. Optimal locations were first selected for a nominal truss, using the linear programming technique previously described. Then, these locations were kept unchanged and control gains as well as structural parameters were tuned to minimize the standard deviation of the most critical constraint.

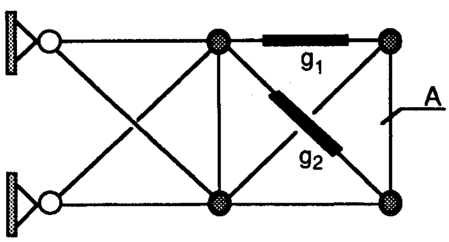

Fig. 2 Ten-bar truss example.

Table 1 Ten-bar truss, optimal designs

	Low weight		High weight	
	($A_{max} = 190 \times 10^{-6}$) $g_1 = 1.036 \times 10^{-4}$ $g_2 = 0.484 \times 10^{-4}$ $A = 131.6 \times 10^{-6}$		($A_{max} = 670 \times 10^{-6}$) $g_1 = 0.961 \times 10^{-4}$ $g_2 = 0.467 \times 10^{-4}$ $A = 667.2 \times 10^{-6}$	
Mode	ζ	$\sigma(\zeta)$	ζ	$\sigma(\zeta)$
1	0.0413	0.00018	0.1200	0.00026
2	0.0335	0.00136	0.0300[a]	0.00013
3	0.0360	0.00106	0.1960	0.00082
4	0.0300[a]	0.00016	0.0833	0.00038

[a]Critical constraint.

Numerical Examples

Active Control of a Ten-Bar Truss

The ten-bar truss shown in Fig. 2 is composed of two square bays, each 9 m wide. Four nonstructural masses of 10 kg each are attached at the four free nodes. The aluminum members have a Young's modulus of 70×10^9 N/m^2 and a specific mass of 3000 kg/m^3. All have the same cross-sectional area with a nominal value of 70×10^{-6} m^2. The undamped natural frequencies of the first four modes are 7.3, 22.0, 23.6, and 42.0 Hz. Two integral force feedback active members are to be included in the truss to provide each of the first four modes with at least 3% relative damping.

In the first stage of the optimization—the selection of active member locations—the gains of the active members were set to a nominal value [$g_1 = g_2 = 10^{-4}$ m/(N-s)] and the linear optimization resulted in the selection of the two members shown in Fig. 2.

Next, the gains of the two active members (g_1, g_2) and the cross-sectional area of the bars (A) (all assumed to have the same area) were adjusted to minimize the sensitivity of the most critical damping ratio to parametric uncertainties. The uncertainty was confined to the magnitudes of the four added masses, which were assumed to be uncorrelated, normal random variables with a coefficient of variation (standard deviation divided by mean value) of 1%. Additional constraints were placed on the maximum values of the gains (limited to five times their nominal values) and on the maximum cross-sectional area. Since in this example the finite element model is of very small size, an exact analysis (full-order model) was used during the optimization to evaluate the damping ratios and their derivatives as well as the standard deviation of the KS function.

Two optimal structure-control designs were obtained by changing the upper limit on the cross-sectional area of the bars (see Table 1). For both designs, the standard deviations of the damping ratios were evaluated from Monte Carlo simulations (50 points). For the low-weight design the damping ratio of the fourth mode was critical (at the 3% limit), while for the high-weight design the second mode's damping ratio was critical. Notice that the standard deviations of the damping ratios vary widely between the two designs, and that the critical damping ratio has, indeed, the lowest standard deviation in both designs. The standard deviation of the damping ratio of the second mode is an indication of the power of the method. The damping ratio of that mode is similar for both designs, but the standard deviation is one-tenth as large when this mode is the critical one: for the low-weight design the coefficient of variation in that mode is about 4%, while for the high-weight design it is about 0.4%. The results indicate that it is indeed possible to confer selective robustness to the most critical measure of the response.

Table 2 Space truss, undamped natural frequencies

Mode	1	2	3	4	5	6	7	8	9	10	11
f_n, Hz	4.7	8.7	9.3	17.5	24.9	32.0	36.4	37.9	53.1	55.2	57.1

Table 3 Space truss, minimum gain, and minimum sensitivity designs

		Minimum gain design		Minimum sensitivity design	
Mode	ζ^R, %	$\bar{\zeta}$, %	$\sigma(\zeta)$, %	$\bar{\zeta}$, %	$\sigma(\zeta)$, %
1	4.09	4.09[a]	0.19	12.29	0.60
2	4.00	4.00[a]	0.40	4.03[a]	0.17
3	2.05	2.38	0.15	8.38	0.74
4	1.09	1.09[a]	0.08	1.13	0.08
5	0.77	0.96	0.05	2.15	0.09
6	0.59	0.61	0.03	1.58	0.07
7	0.53	0.53[a]	0.17	1.51	0.08
8	0.51	0.51[a]	0.16	1.10	0.09
9	0.36	0.40	0.04	0.99	0.10
10	0.35	0.43	0.03	1.04	0.08
11	0.33	0.41	0.02	0.74	0.04

[a]Critical constraint.

Active Control of a Space Truss

The space truss shown in Fig. 3 has 207 members and 71 nodes. Six masses (representing about 40% of the total mass of the structure) have been added to create closely spaced frequencies. The natural frequencies of the first 11 modes of the undamped structure are listed in Table 2.

In the first phase of the design we optimized the locations of eight active members with nominal gains to maximize the decay rate of the first 11 natural vibration modes of the truss. All 207 members were considered candidate locations for the active members.

The decay rate of a vibration mode is the product of its damping ratio multiplied by its undamped natural frequency. Using the approximation (17) for the damping ratio we have

$$\omega_k \zeta_k \approx \frac{1}{2} \sum_{i=1}^{8} g_i \nu_{ik} k_{a_i} \tag{20}$$

The decay rate has been chosen as the objective instead of the damping ratio because of the larger range of frequencies of the target modes. Uniform damping ratios would give a very fast decay to high-frequency modes and much slower decay to low-frequency modes. The linear programming formulation of Eq. (18) is slightly modified to maximize the decay rate as

$$\text{Find } e_i,\ i = 1, \ldots, 207 \text{ and } \beta$$

$$\text{to maximize } \beta$$

$$\text{such that } \omega_k \zeta_k = \frac{1}{2} \sum_{i=1}^{207} g_i e_i \nu_{ik} k_{a_i} \geq \beta$$

$$\text{and } \sum_{i=1}^{207} e_i = 8 \tag{21}$$

The resulting locations are shown in Fig. 3.

The second phase consists of minimizing the sensitivity of critical modes to parametric uncertainties by adjusting the magnitudes of the six masses and the gains of the eight active members. The uncertainties are limited to the magnitudes of the six nonstructural masses and the gains of the eight active members. All are assumed to be uncorrelated normal random variables with coefficients of variation (standard deviations divided by mean values) of 10% for the gains and 5% for the masses. The requirements on the damping ratios correspond to a decay rate of 1.2 s^{-1} on all but the second mode. The requirement on the second mode was increased to 4% relative damping (see Table 3) so as to make it a critical mode. The Ritz technique mentioned before (also see Ref. 20) was used to evaluate the damping ratios and their derivatives. The masses are constrained to vary within 30% of their nominal value to limit the errors in the approximate eigenanalysis. The gains are allowed to vary between zero and 10 times their nominal values. The optimum design obtained with this formulation will be referred to as the minimum sensitivity design.

For comparison, we generated a baseline minimum gain design subject to the same requirements. For this, we changed the objective function to minimize the sum of the squares of the gains of the eight active members, i.e.,

$$\min \sum_{i=1}^{8} g_i^2 \tag{22}$$

using the same design variables (six masses and eight gains). The resulting design will be referred to as minimum gain design.

The estimated average values $\bar{\zeta}$ and standard deviations $\sigma(\zeta)$ of the damping ratios of the two designs are given in Table 3.

Figure 4 compares graphically the performance of the two designs. The horizontal axis shows the mode number, and the vertical axis gives the value of the damping ratio of each mode, normalized by the corresponding required damping ratio. The horizontal line at $\zeta_k/\zeta_k^R = 1$ represents the required levels of damping. The bars represent the range of damping ratios corresponding to the nominal (average) value plus and minus three standard deviations. Using this design, the damping ratios of an actual structure would have a 99.7% probability of falling inside these boxes.

As seen in Fig. 4, the baseline design is unacceptably sensitive to the uncertainties. This is the result of two factors: first, almost all modes are critical or close to critical, so that any deviation from the nominal structure is likely to pull the damping ratios below the required values; and second, the sensitivities of some critical damping ratios (modes 2, 4, 7, and 8 in particular) are large so that uncertainties are likely to create large violations of the requirements.

The minimum sensitivity design on the other hand, is far better protected from the effects of uncertainties. The optimizer has made most modes noncritical by increasing the nominal values of their damping ratios (modes 7 and 8 in particular). Mode 2 has been made less sensitive to uncertainties as can be seen from the reduced value of its standard deviation. This protection has in effect been targeted on the critical mode in the final design (i.e., mode 2). The design has

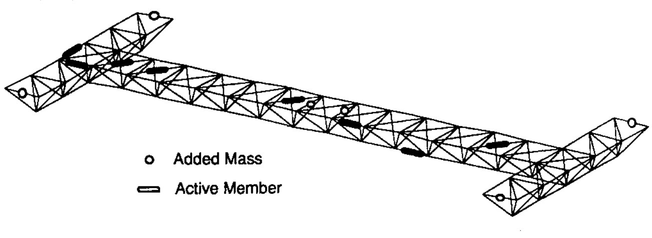

Fig. 3 Three-dimensional truss, active member and mass locations.

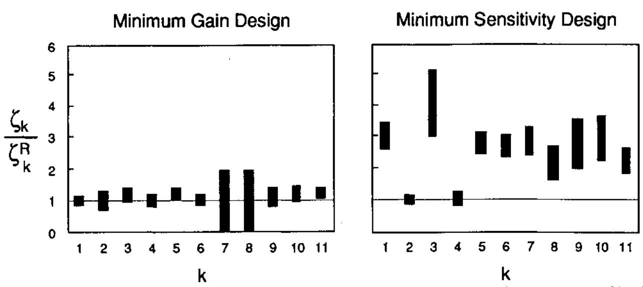

Fig. 4 $[\bar{\zeta}_k \pm 3\sigma(\zeta_k)]/\zeta_k^R$, $k = 1, \ldots, 11$ for minimum gain and minimum sensitivity designs.

been tailored in a way that favors critical modes at the expense of less critical modes. The sensitivities of the damping ratios of modes 1 and 3, for example, have increased, but their average value has been pushed away from the constraint limit so that the increased sensitivity is not likely to produce constraint violations. The net result of the optimization is a reduced probability for the design to violate performance requirements because of uncertainties in physical properties of the actual structure.

Conclusions

A new method for targeted robustness optimization has been presented. This method allows the analyst to target the most critical performance parameter for enhanced protection from uncertainties in the model. The variance of the KS function was successfully used as an estimator of the variance of the most critical constraint. The power of the targeted robustness method was demonstrated numerically using two active damping truss examples. The results showed that the most critical damping ratio was selectively protected—its sensitivity to the uncertainties was substantially below those of noncritical damping ratios.

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